



Allowed Tables and Charts: *None*

Please, Answer all the following Questions

**Question (1) (28 Marks)**

- (a) What is a Servomechanism as a control term (one illustrated example)? (4 Marks)  
(b) When a control system is described as a linier system?. (4 Marks)  
(c) The closed-loop transfer function of a control system is given by:

$$\frac{C(s)}{R(s)} = \frac{1}{(s^3 + b_1s^2 + b_2s + b_3)}$$
, the Hurwitz determinants are computed as  $D_1 = 6$ ,  $D_2 = 60$ , and

$$D_3 = 360.$$

- (i) Evaluate the coefficients  $b_1$ ,  $b_2$ , and  $b_3$ . (4 Marks)  
(ii) Find the unit step response of the system; i.e. final value of  $c$  if  $R(s) = \frac{1}{s}$ . (3 Marks)  
(iii) Knowing that one of the closed-loop poles = -1, find the other two poles. (3 Marks)  
(iv) Find  $c(t)$ , if the system is subjected to an impulse input; i.e.  $R(s) = 1$ . (4 Marks)

Hint: if  $F(s) = \frac{1}{s+a}$ , then  $f(t) = e^{-at}$

- (d) A feedback control system has a characteristic equation:  $s^3 + (4+K)s^2 + 6s + 16 + 8K = 0$   
The parameter,  $K$ , must be positive. Using Routh's stability table, evaluate the maximum value of  $K$  that can be tuned before the system becomes unstable. When  $K$  is equal to the maximum value, the system oscillates. Determine the frequency of oscillation. (6 Marks)

**Question (2) (20 Marks)**

- (a) Which properties that we may get when both polar and Bode plots are drawn for any control system? (4 Marks)  
(b) Draw Polar Plot for a plant transfer function  $G(s) = \frac{30}{s(1+3s)(1+5s)}$  after computing intersection of the curve and the vertical asymptote with real axis. (8 Marks)  
(c) Draw Bode Plots for  $G(s)$ , then detect both gain- and phase margins graphically.

Hint: for  $F(j\omega) = \frac{1}{1+j\tau\omega}$ , you may obtain  $\phi = -\tan^{-1}(\tau\omega)$ . (8 Marks)

**Question (3) (22 Marks)**

- (a) Discuss briefly the benefits of introducing a feedback (four only) to a control system? (4 Marks)  
(b) It is required to place the poles of a system using a series compensator (controller),  $G_c(s)$ , and the Diophantine equation for the plant  $G_p(s) = \frac{s+3}{s^3+s^2+10}$ . The desired closed-loop poles are -3, -6, and as many factors of  $(s+10)$  as you need to the desired pole polynomial to find a proper controller.

(18 Marks)

P. T.O. →



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**Question (4) (20 Marks)**

(a) Define the controllability of a control system; explain how it can be used in transforming state-variable representation of a control system from one form to another. (5 Marks)

(b) The input-output transfer function of a plant  $G_p(s) = \frac{Y(s)}{U(s)} = \frac{4}{s^3 + 5s^2 + 4}$  may be represented into physical state-variable form as:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u, \text{ and } y = [1 \ 0 \ 0] x$$

- (i) Deduce the phase-variable representation of the given system. (6 Marks)  
(ii) compute the controllability matrices for both representations; hence, compute the transformation matrix,  $P_p$ , between both forms based on these controllability matrices. (6 Marks)

(iii) Design the state-variable feedback controller gain,  $k^T$ , with forward gain,  $K=1$ , that can be used to place the poles of the closed-loop system at  $s=-2, -3$ , and  $-10$  (3 Marks)

**NOTE:** For plant description in phase-variable form, the appropriate values of  $k_p$  is easily found from the relation  $k_p = (d-a)/K$ . Where  $d$  is the column vector of coefficients of the desired  $n$ <sup>th</sup>-order polynomial that the closed-loop poles to be located. While  $(a)$  is the vector of coefficients of characteristic equation of the plant,  $k^T = k_p^T * P_p$

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**G O O D L U C K**